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Optimal planning for optical transport networks

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In optical transport networks, recent development of new technologies has led to highly accelerated ('disruptive') increases in the capacity associated with a given investment cost. As a result, there have been dramatic decreases in the cost per unit of transport. We describe a nonlinear mixed-integer planning model that assumes both the continuous emergence of new systems and a constant-elasticity demand function. Optimization of the model with respect to price and technology acquisitions over time suggests that, with high elasticity and steeply dropping technology costs, a carrier will maximize net present value by frequently deploying new systems. This conclusion is in sharp contrast to the analogous results for voice networks, where demand is much less elastic and the rate of technology change is much slower.

Keywords: optical transport networks; network planning; optimization; economic modelling

1. Introduction

Planning for future network expansion is a complex problem for communications carriers today. Rapid technology innovations are constantly changing the costs of network expansion; not only are existing systems decreasing in price every year, but more advanced systems are introduced at frequent intervals. With a modest increase in investment costs, the newer systems provide capacity that is orders of magnitude larger than in earlier systems, thus dramatically reducing the cost per unit of capacity. These cost trends create a major trade-off in making network deployment decisions: rapid deployment of current systems allows a carrier to collect revenue in the short term, but may rule out future opportunities to exploit cost savings. Two further challenges in planning are that price reductions lead to nonlinear increases in demand, and that past, current and future deployment decisions are highly interdependent. All of these connections generate a labyrinth of alternatives that need to be evaluated to make good planning decisions.

We examine these issues through a model that optimizes net present value over time for a carrier in long-haul transport networks, where innovations are currently driven primarily by the development of synchronous optical network (SONET) ring and wavelength division multiplexing (WDM) technologies. An important element in our model is its inclusion of the relationship between price and demand. If there is a highly elastic response of demand to price, the carrier's best strategy is to reduce price as technological progress provides larger and more cost-effective systems, in order to increase revenue. The more elastic demand is, the greater is the increase in revenue that results from price reductions afforded by falling unit costs. These revenue opportunities influence the carrier's incentive to invest in more capacity. Because of these interdependencies, we believe that the carrier's strategy should not be based on demand that is specified externally, but rather should be a coupled optimization of prices and capacity acquisition over time. Assuming what we believe to be reasonable demand elasticity, our conclusion is that carriers will maximize net present value by frequent network expansion.

This paper is organized as follows. Section 2 discusses current technologies in optical transport networks, § 3 describes and justifies a constant-elasticity demand curve, § 4 presents our model, § 5 analyses results and § 6 summarizes conclusions and discusses directions for future work.

2. Optical ring transport network technologies

Optical transport networks are designed to carry communications traffic between cities typically separated by hundreds or even thousands of miles. Cities are connected by fibre cables into a ring architecture, and an optical transmission system is deployed at each city on that ring. Amplifiers and regenerators are needed along each fibre route connecting adjacent cities, where the equipment spacing depends on the technology.

Because the signal is transmitted along the fibre in SONET format (Stern & Bala 1999), a connected ring of optical transmission systems is usually called a SONET ring. There are two major types of SONET rings: unidirectional path-switched rings (UPSR) and bidirectional line-switched rings (BLSR). With UPSR, transmission systems are connected by two fibres, one each for clockwise and counterclockwise transmissions. With complete redundancy, the ring capacity of a UPSR system should equal the sum of traffic between all city pairs.

With BLSR, transmission systems are connected by four fibres, two that transmit along clockwise and counterclockwise directions as with UPSR, and two that serve as backup. Although a BLSR system is more expensive because it requires more sophisticated add–drop multiplexers (ADMs) as well as an additional fibre pair, it can carry more traffic than UPSR by intelligent assignment of traffic on different segments of the ring. With the exception of routing, planning processes are the same for both UPSR and BLSR; hence, for simplicity, discussion in this paper is based on UPSR.

It was formerly customary to deploy a SONET ADM in each city for each fibre. ADM systems transform electronic to optical signals, and transmit the signals onto the fibre; they also intercept optical signals from the fibre and transform them into electronic form. The capacity of a given ring is determined by the ADM rates defined in the SONET standard, with OC-1, the base rate, defined as 51.84 Mb s^{-1}. The most commonly used capacity today is OC-48, which is equivalent to 2.5 Gb s^{-1}. However, the adoption of OC-192 (10 Gb s^{-1}) has begun, and OC-768 (40 Gb s^{-1}) is on the horizon.

In recent years, optical transport networks have been revolutionized by the advent of WDM systems, which allow optical signals to be transmitted on different wavelengths within one fibre. As a result, rather than dedicating a fibre pair to each individual ADM, multiple ADMs may be attached to one WDM terminal, and several WDM terminals at different sites can be connected by a single fibre pair.

With WDM, there are dramatic savings in the costs of fibre, amplifiers and regenerators, which are a major component of the capital expense. In addition, the amount of traffic that can be carried on a single fibre pair is no longer restricted by the ADM transmission rate. Instead, the maximum capacity can reach the product of the ADM rate and the maximum number of wavelengths accommodated by WDM. For example, an ADM-only OC-48 SONET ring can provide at most 2.5 Gb s^{-1} on a pair of fibres, but a SONET ring equipped with a 40-wavelength WDM system can carry 100 Gb s^{-1}.

3. Estimation of demand

(*a*) *Formulation of the demand function*

Realistic planning in an environment of rapid gains in technology depends on an appropriate representation of the relationship between demand and price. However, it is not easy to characterize this relationship. Because the communications industry is experiencing an unprecedented transformation, past experience with the service market becomes less convincing as an indicator of future demand. The most well-known example of this difficulty is usage of the Internet (Cerf 1998), whose growth in capacity has led to the creation of many previously unprofitable applications, in turn creating more demand for capacity. The most optimistic extrapolations have consistently underpredicted the continuing expansion of the World Wide Web, and there is no indication of any change in this trend.

Similarly, the famous 'Moore's law', which dominates thinking about the microprocessor and dynamic random access memory (DRAM) industries, is sometimes stated for the former as a doubling in speed every 18 months for the same price. However, what actually happens is not that the same chips are available 18 months later for half the price, but that different chips, with twice the speed, are available at the same price. The availability of more computing speed thus generates not only an 'upgrade' in available speed, but also new markets, since applications that were formerly impractical are enabled by the faster chips. The net effect is that the demand for chips more than doubles every 18 months, i.e. remains ahead of the growth in processor speed.

To capture these properties in modelling demand for optical network capacity, we begin with the concept of elasticity. Let D denote demand and p denote price, where by assumption $p > 0$. Consider D as a function of price. The *price elasticity of demand* E associated with a time-interval is defined as the negative ratio of the relative change in demand and the relative change in price during that interval, i.e.

$$E = -\frac{\Delta D/D}{\Delta p/p}, \tag{3.1}$$

where ΔD denotes the change in demand and Δp denotes the change in price. In the (usual) situation when demand increases as price decreases, the value of E defined by (3.1) is positive. Rearrangement of (3.1) gives

$$\frac{\Delta D}{D} = -E\left(\frac{\Delta p}{p}\right), \tag{3.2}$$

which reveals the implications of different ranges for E. If $E > 1$, it follows from (3.2) that any relative reduction in price leads to a larger relative increase in demand, i.e. if price is reduced by 1% ($\Delta p/p = -0.01$), then demand increases by more than 1%, i.e. $\Delta D/D > 0.01$. If, on the other hand, $E < 1$, a price reduction of 1% leads to an increase in demand of less than 1%.

Taking the limit of (3.1) as the interval of change in price becomes infinitesimal, we obtain

$$E = -D'(p/D). \tag{3.3}$$

Assuming that revenue, denoted by R, is the product of price and demand, i.e. $R = pD$, differentiation with respect to p and manipulation of (3.3) give an expression for the relative change in revenue,

$$\frac{R'}{R} = -\frac{E-1}{p}, \tag{3.4}$$

which shows the following.

(i) If $E > 1$, a reduction in price leads to an increase in revenue—for example, if $E = 1.5$ and price is reduced by 2%, revenue increases by 1%.

(ii) If $E = 1$, revenue is unaffected by any changes in price.

(iii) If $E < 1$, a reduction in price leads to a reduction in revenue—for example, if $E = 0.5$ and price is reduced by 2%, revenue decreases by 1%.

We formulate a demand function with *constant* elasticity by assuming that (3.3) holds with the same value of E for all p and D of interest. Solving the associated differential equation gives the general form

$$D = Ap^{-E}. \tag{3.5}$$

The scaling constant A is equal to the value of D when $p = 1$, so that A can be interpreted as a demand potential. Based on the analysis following (3.4), we assume that $E > 1$. It follows from (3.5) that

$$\frac{D_2}{D_1} = \left(\frac{p_1}{p_2}\right)^E, \quad \text{so that } E = \frac{\ln(D_2/D_1)}{\ln(p_1/p_2)}. \tag{3.6}$$

In this paper, we use the constant value of elasticity for the entire range of price. We are aware that there is a considerable body of prior work devoted to the estimation of elasticity (see, for example, Bass 1980; Greene 1993).

Figure 1 depicts constant-elasticity fits to historical data for demand, with log(units) on both axes. (*a*) In DRAM, price per bit is plotted against available units from 1965 to 1992 (R. Janow 1999, personal communication). (*b*) For electricity, price per unit is plotted against generated electricity from 1926 to 1970 (O'Donnell 1973).

In the DRAM market, a constant elasticity of 1.5 fits the data well. For electricity, the fit with constant elasticity of 2.2 matches the quality of the fit for DRAM in the years when the time-series overlap.

Both of these examples illustrate markets in which new applications arise as capacity increases. Rather than devalue the industry, innovations that allow steep drops in price *increase* the value of the market because the demand response is highly elastic (see (i) following equation(3.4)). The demand curve shifts outward with new

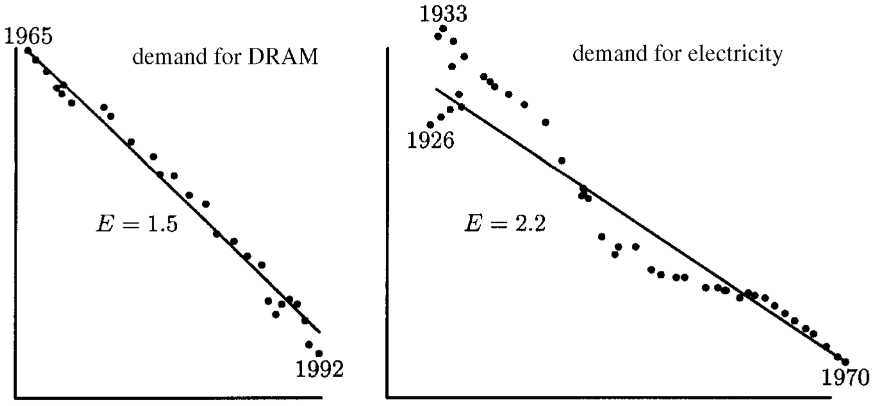

Figure 1. Demand for DRAM and electricity.

applications, each of which has its own product life that can be represented by an 'S'-shaped 'Bass curve' (Bass 1969, 1980). However, their composite is well described by a constant-elasticity curve, so that (3.5) represents a reasonable functional form for long-run demand. Because our intent is to model bandwidth capacity, the constant-elasticity form is both sensible and appropriate; however, our framework is sufficiently general to accommodate other functional forms as needed.

(*b*) *Estimation of demand elasticity*

A suitable range of values for E in (3.5) can be estimated directly and indirectly.

(i) *Direct measurement*

The Optical Network Business Unit of Lucent Technologies estimates (Optical Network Business Unit, Lucent Technologies 1999, personal communication) that optical equipment is doubling in capacity for the same dollar cost every year, so that it is, in effect, possible to offer the same capacity for half the price. Furthermore, traffic (i.e. demand) is forecast to be between two and three times larger each year. Applying (3.6) with $p_1/p_2 = 2$ and $D_2/D_1 = 3$ gives $E = 1.59$; if $p_1/p_2 = 2$ and $D_2/D_1 = 2$, equation (3.6) shows that $E = 1$.

The traditional elasticity estimate for voice traffic is approximately 1.05, and France Telecom has recently estimated elasticity as 1.337 (Aldebert *et al.* 1999).

(ii) *Derived measure*

The derived measure of elasticity is based on the notion that elasticity of demand is the same for both equipment and service providers if equipment providers have strong market power. Table 1 shows the values of elasticity for industry-wide demand for bandwidth capacity in various classes of equipment. There is innovation in several equipment categories, including transmission (WDM and usable frequency windows in fibre) and communications software to reduce support costs. Making the correspondence between voice service and digital circuit switch, noting the higher elasticity for asynchronous transfer mode (ATM) equipment, we infer that the elasticity for data service is correspondingly higher.

Table 1. *Demand elasticity*

equipment	estimated elasticity	original source
digital circuit switch	1.28	NBI
WAN ATM core switch	2.84	In-Stat, IDC
LAN ATM backbone switch (ForeRunner ASX-200BX, small ASX-1000)	2.76	Dell'Oro
WAN ATM edge switch	2.11	In-Stat, IDC

4. Assumptions and the model

Our model is intended to maximize net present value by choosing optimal prices and technology purchases.

(a) *The basic formulation*

Suppose that there are N cities, with a set $\mathcal{I}$ of indices of (distinct) city pairs, and that we are interested in time, year 1 through T.

(i) *Revenue*

As discussed in § 3 *a*, demand between city pairs is assumed to satisfy (3.5) with a constant elasticity E so that, for any city pair $(i,j) \in \mathcal{I}$ in period t, demand and price satisfy

$$D_{ijt} = A_{ijt} p_{ijt}^{-E}, \tag{4.1}$$

where D_{ijt} is demand (measured in OC-1 units) between cities i and j, p_{ijt} is the annual price per OC-1 between cities i and j, and A_{ijt} is the scaling factor relating them (see equation (3.5)). The total revenue in period t is then given by

$$R_t = \sum_{(i,j)\in\mathcal{I}} p_{ijt} D_{ijt}. \tag{4.2}$$

(ii) *Costs*

In formulating the cost of operating the network, we ignore investment and expenses that have few direct effects on ring deployment and pricing decisions, and we assume that the costs of building conduits and laying fibre cables are sunk. The two major cost components represented in the model are as follows.

(1) An initial one-time investment cost for the purchase and installation of hardware, such as optical transmission units, WDM terminals and regeneration and amplification equipment.

(2) Recurring maintenance costs for fibre.

The set K is defined as the set of WDM technologies. For each technology $k \in K$, τ_k denotes the time period in which this technology first becomes available and κ_k denotes the maximum capacity (in OC-1) of a single system in technology k.

For each technology k and period t, I_{kt} denotes the acquisition cost of a WDM system and m_{kt} denotes the per-mile cost of maintaining each fibre route. For each two-fibre UPSR ring of technology k purchased in period t, the investment cost is NI_{kt}; for each ring operated in period t, the recurring expense is $2Lm_{kt}$, where L denotes the total length of fibre needed to connect all the cities.

The number of WDM systems of technology k bought in period t is denoted by b_{kt} ('b' for 'bought'), and the number of such systems used is denoted by u_{kt} ('u' for 'used'). The expense in period t is then

$$\text{Expense}_t = N \sum_{k\in K} I_{kt} b_{kt} + 2L \sum_{k\in K} m_{kt} u_{kt}. \tag{4.3}$$

(iii) *Net present value*

Combining (4.2) and (4.3), the cash flow C_t in period t is

$$C_t = R_t - \text{Expense}_t. \tag{4.4}$$

In addition to considering cash flow, our model includes a terminal value function that depends on two parameters: a (perpetual) growth rate in cash flows, called g_∞, and an integer f satisfying $0 \leqslant f < \infty$ (see Lanning *et al*. (2000) for details). Letting $\delta = \rho + g_\infty$, where ρ is the discount rate, terminal value is defined by

$$\text{TV} = \begin{cases} \dfrac{1-\delta^f}{1-\delta} C_T & \text{if } \delta \neq 1, \\ fC_T & \text{if } \delta = 1, \end{cases} \tag{4.5}$$

where C_T is the cash flow in the final period (see equation (4.4)).

The net present value over periods 1 through T is given by

$$\text{NPV} = \sum_{t=1}^{T} C_t \rho^t + \rho^{T+1}\,\text{TV}, \tag{4.6}$$

where ρ denotes the assumed discount rate (with $\rho < 1$). The objective of the model is to maximize NPV (i.e. discounted cash flow) as defined by (4.6).

(iv) *Constraints*

The following constraints are imposed in each time period.

(1) Total demand between cities i and j cannot exceed the capacity provided by all available systems:

$$\sum_{i,j\in\mathcal{I}} D_{ijt} \leqslant \sum_{k\in K} \kappa_k u_{kt}.$$

(2) The number of systems used cannot exceed the number used in the previous time period plus the number bought in the current period:

$$u_{kt} \leqslant b_{kt} + u_{k,t-1}.$$

In effect, this constraint states that, once a system has been 'retired', i.e. has ceased to be used, it cannot be used later. In addition, the number of systems used in period 1 must be equal to the number bought in that period, so that $u_{k1} = b_{k1}$ for all technologies k.

(3) No system of technology k can be bought before that technology becomes available:

$$b_{kt} = 0, \quad t < \tau_k, \quad \text{for all } k.$$

(4) All prices and the numbers of systems bought and used must be non-negative.

(v) *The optimization problem*

The parameters to be optimized are the city-pair prices p_{ijt} and the numbers of systems of each technology bought and used, b_{kt} and u_{kt}. Putting together the objective and constraints, the problem is

$$\underset{p,u,b}{\text{maximize}}\, \text{NPV} \tag{4.7}$$

subject to

$$\begin{aligned}
\sum_{i,j\in\mathcal{I}} D_{ijt} &\leqslant \sum_{k\in K} \kappa_k u_{kt}, && t = 1,\dots,T\\
u_{kt} &\leqslant b_{kt} + u_{k,t-1}, && k \in K, \quad t = 2,\dots,T\\
b_{kt} &= 0, && t < \tau_k, \quad k \in K, \quad t = 1,\dots,T\\
p_{ijt} &\geqslant 0, && (i,j) \in \mathcal{I}, \quad t = 1,\dots,T\\
u_{k1} &= b_{k1}, && k \in K\\
b_{kt} &\geqslant 0, && k \in K, \quad t = 1,\dots,T\\
u_{kt} &\geqslant 0, && k \in K, \quad t = 1,\dots,T
\end{aligned}$$

where NPV is defined by (4.6), $p = \{p_{ijt}\}$, $u = \{u_{kt}\}$ and $b = \{b_{kt}\}$.

(*b*) *Representing technology innovations*

Optical transport networks are characterized by frequent, substantial increases in capacity. To parameterize this phenomenon in the model, we assume that each new WDM technology has capacity μ times larger than that of its immediate predecessor, i.e.

$$\kappa_k = \mu \kappa_{k-1}, \tag{4.8}$$

where $\mu > 1$.

To characterize the effects of a new technology on costs, d ('d' for 'disruptiveness') is defined as the reduction in initial investment cost per unit of capacity in technology k compared with technology $k-1$, so that the per-unit investment costs satisfy

$$\frac{I_{k\tau_k}}{\kappa_k} = (1-d)\frac{I_{k-1,\tau_{k-1}}}{\kappa_{k-1}}. \tag{4.9}$$

For example, $d = 0.2$ means that the per-unit cost of a new technology is 80% of the per-unit cost of the preceding technology. The larger the value of d, the greater reduction in the per-unit investment cost, i.e. the greater the disruptiveness. For simplicity, d is assumed to be constant; in a more complex model, d could depend on time and/or technology.

Finally, we assume that, once a technology has become available, its investment cost decreases by the (constant) factor η, i.e.

$$I_{k,t+1} = \eta I_{kt} \quad \text{for } t \geqslant \tau_k. \tag{4.10}$$

(c) *Optimizing the model*

Because the price variables p_{ijt} appear nonlinearly in the formulation of demand (see (4.1)), the optimization problem (4.7) has a nonlinear objective function and nonlinear constraints. Furthermore, the numbers of systems bought and used must be integers. Unfortunately, there are no currently available algorithms, let alone general-purpose software, for solving a general mixed-integer nonlinear optimization problem.

Our approach to solving (4.7) is a sequential continuous relaxation technique, an approach widely used in optimization (see, for example, Fletcher 1987, Gill *et al*. 1981). For details of its application to our model, see Lanning *et al*. (2000).

5. Analysis and results

The purpose of developing this model is to explore, to low order, the effects of both price elasticity and changes in per-unit technology costs.

We give results for a specific problem: a hypothetical five-city ring network in which the distance between adjacent cities is 500 miles. Thus N is 5, there are 10 city pairs, and L, the length of fibre needed to connect the cities, is 2500. Demand is scaled by assuming an initial demand of 10 OC-1 between each city pair, with an initial monthly price of $18 000 per OC-1; these values determine the coefficients A_{ijt} in equation (4.1).

Fibre maintenance costs are estimated using the Hatfield model (HAI Consulting Inc. 1998). The life of a conduit is assumed to be 30 years, with eight-year depreciation of the fibre cables. Consequently, the amortized yearly investment cost of laying a conduit and installing a fibre cable ranges from $172 to $516 per mile per fibre, depending on cable size, which ranges between 24 and 96 fibres per cable. It can also be derived from the Hatfield model that the ratio of amortized investment expense to recurring supporting expense is around $1/0.3 \approx 3.33$. By applying this ratio, yearly fibre maintenance expense is estimated to be between $52 and $155 per mile per fibre. For simplicity, we use the constant value $m_{kt} = \$100$ per mile per fibre in equation (4.3) for all technologies and time periods.

Each technology is represented by the data rate of ADM and the number of wavelengths of WDM. We assume the technology available in period 1 is OC 48-40, and the next five technologies are OC 48-80, OC 192-40, OC 192-80, OC 768-40 and OC 768-80, each entering in the subsequent time period. Therefore, the factor μ defining the growth in capacity in (4.8) is taken as 2, meaning that each new technology has double the capacity of the preceding one. While there can be other paths of technology evolution, our result is more sensitive to d, the reduction of investment

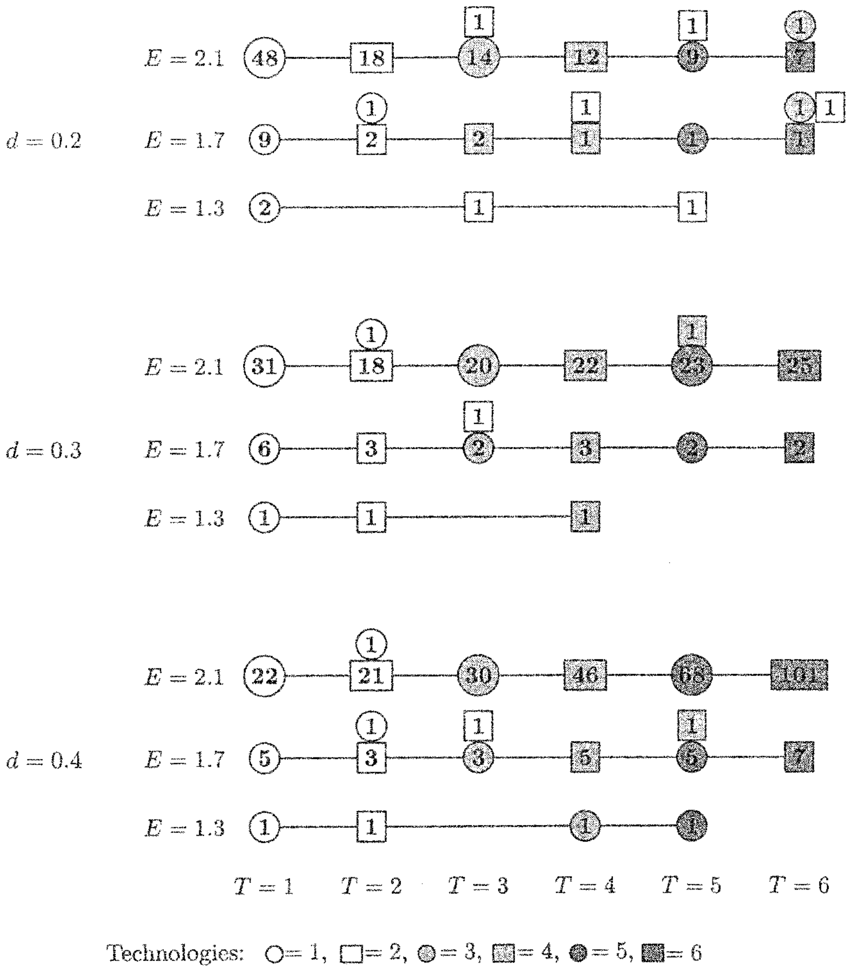

Figure 2. Technology acquisitions over time.

cost per unit of capacity, than to μ, the size of capacity increment of different technologies. It has been estimated that, in transport networks, the saving in unit cost from the next-generation technology is *ca.* 28% (Crowe 2000). For this reason, we set d to 30% and vary it by $\pm 10\%$, and examine how the results change.

In period 1, the investment cost per OC-1 is assumed to be \$2500, so that the per-system cost for technology 1, I_{11}, is \$4 800 000. Thereafter, the per-system cost I_{kt} in (4.3) is defined by (4.8), (4.9) and (4.10). For a fixed technology, the yearly reduction of investment cost should be smaller than the saving from a next-generation technology (30%). Therefore, the parameter η specifying the per-period reduction in investment cost for a fixed technology in (4.10) is taken as 0.9, i.e. the investment cost for a fixed technology decreases by 10% in each period. The discount rate ρ, used in (4.6), is 0.86. In computing the terminal value (see (4.5)), we take $g_\infty = 0.07$ and $f = 7$.

The generation and number of technology acquisitions in periods 1 through 6 are shown in figure 2, where the integer inside the symbol for each technology indicates

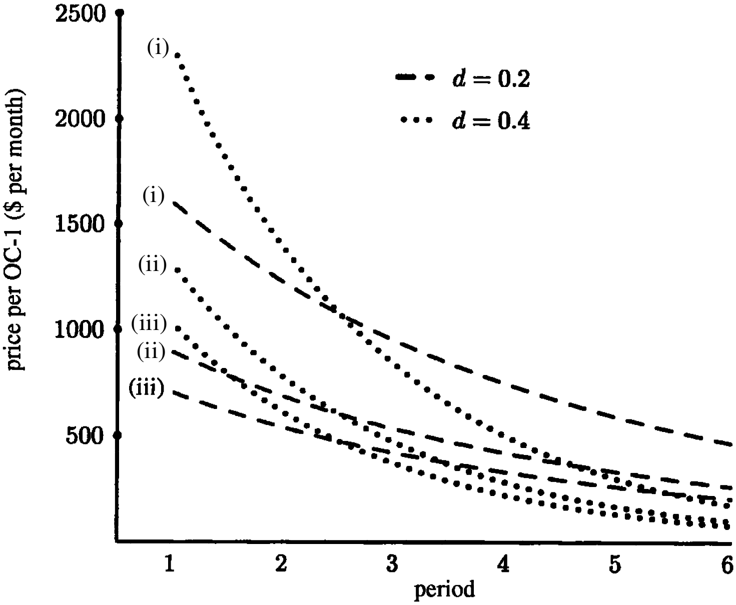

Figure 3. Prices over time. The three values of elasticity are (i) 1.3, (ii) 1.7 and (iii) 2.1.

the number of such systems purchased in the given time period. To avoid dependence on the definition of terminal value, the variables have been optimized over a longer time horizon so that the results for periods 1 through 6 do not change as the time horizon increases; for the results given, this corresponds to $T = 10$.

The results in figure 2 correspond to three values of the disruptiveness factor, $d = 0.2$, $d = 0.3$ and $d = 0.4$ (see (4.9)). A larger value of d means more steeply dropping per-unit technology costs, so that (for example) $d = 0.3$ means a 30% per-unit cost reduction for each new technology.

For each value of d, technology acquisitions are shown for three values of elasticity, ranging from mildly elastic ($E = 1.3$) to highly elastic ($E = 2.1$). For a fixed disruptiveness, figure 2 shows the following.

(i) Larger elasticity implies more innovative technology acquisitions, i.e. new technologies tend to be acquired sooner.

(ii) The number of systems acquired increases with larger elasticity.

(iii) Technology acquisitions occur more frequently as elasticity increases. With an elasticity of 1.3, even with high disruptiveness ($d = 0.4$), only one system is acquired in any given period.

Figure 2 also allows us to compare the effects of disruptiveness on technology acquisitions. For a fixed elasticity, higher disruptiveness, which means a greater reduction in per-unit cost over time, leads to acquisition of less equipment in the early time periods and more in the later time periods. In period 1, with elasticity 2.1, 48 systems are acquired when d is 0.2, 31 systems when d is 0.3, and 22 systems when d is 0.4. In period 6, by contrast, eight systems (seven of technology 6, one of technology 3) are acquired when d is 0.2, 25 systems (all of technology 6) when d is 0.3, and 101 systems (all of technology 6) when d is 0.4.

Our model provides insight into the effects of elasticity and disruptiveness on the optimal pricing scenario. Figure 3 shows the prices over time for two values of disruptiveness ($d = 0.2$ and $d = 0.4$) and three values of elasticity (1.3, 1.7 and 2.1).

An obvious conclusion that can be drawn from figure 3 is that, for a fixed disruptiveness, prices are uniformly lower across all time periods as elasticity increases. This result reflects the observations of § 3 that higher elasticity implies higher revenue from price reductions.

A second implication of figure 3 is that, for a given elasticity, a larger disruptiveness means a higher *initial* price but a lower price in later periods (period 3 and after).

Finally, our model reveals the relationship between elasticity, disruptiveness and the growth in capacity of the network over time. Figure 4 shows capacity (on a log scale) as a function of time for the same two values of disruptiveness ($d = 0.2$ and $d = 0.4$) and three values of elasticity (1.3, 1.7 and 2.1) shown in figure 2. Observe that, for a given elasticity, the *initial capacity* is lower for larger disruptiveness, but that capacity thereafter grows much more rapidly for the larger disruptiveness. Similarly, for a given disruptiveness, a larger value of elasticity implies a larger capacity in every time period. For the largest values of elasticity ($E = 2.1$) and disruptiveness ($d = 0.4$), capacity increases by a factor of more than 200 during the six time periods shown.

6. Conclusions

We have developed a model to analyse the optimal growth of optical transport networks. Unlike previous planning models (Freidenfelds 1981) that view future demand as externally given, we allow demand to be determined by prices via a constant-elasticity demand function; prices are then optimized jointly with capacity investment decisions. The formulation of a constant-elasticity demand function represents current communications technology trends reasonably well. Our results show that frequent deployment of newer systems is optimal in an environment of high elasticity and large reductions in the per-unit cost of technology.

These results run counter to traditional voice network carrier practice, where expansion of networks is slow and the timing of investments is irregular. The difference can be attributed to inclusion of high elasticity and disruptiveness, which are positively correlated with aggressive deployment of newer systems.

These results demonstrate the range over which frequent investments make sense and how the rate of investment may accelerate or decelerate depending on the level of technology disruptiveness. With appropriate assumptions, we can map forecasts of capacity growth to a combination of technology disruptiveness and elasticity. The average exponential capacity growth rates that we estimate range between 30 and 90% per year. While these average growth rates are high, they pale in comparison with the 200–300% growth rates sometimes reported by industry consultants and analysts. Forecasts of higher rates of growth may be justified by other factors. One possible factor is that regulation has retarded investment. Less regulation or the entry of unregulated firms affords the industry a one-time adjustment to the optimal expansion path, which is reflected as a period of growth at the highest rate that firms can accommodate and that the market can absorb. Further work is needed to

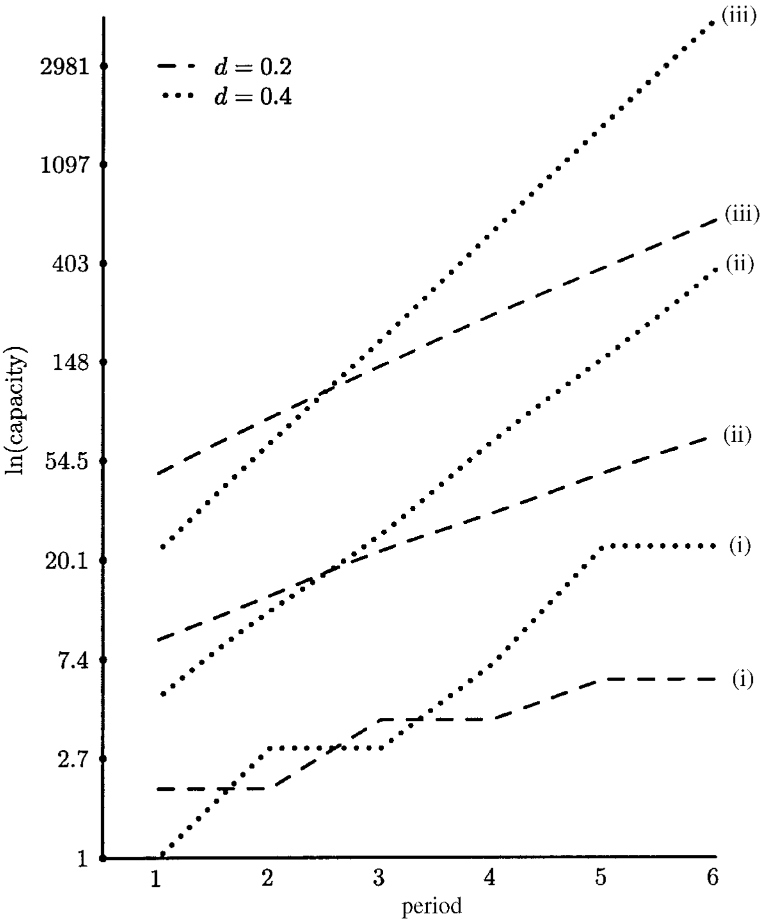

Figure 4. Capacity (shown on a log scale) over time. The three values of elasticity are (i) 1.3, (ii) 1.7 and (iii) 2.1.

extend this method from a monopoly or cooperative solution to more competitive solutions.

Future modelling work will move in several directions. First, random factors can be included to account for uncertainties in both future demand and technology environments. Another direction is to introduce competitive carriers for which a Cournot capacity choice model (Herk 1993) is used to replace the current maximization of NPV.

References

Aldebert, M., Ivaldi, M. & Roucolle, C. 1999 Telecommunication demand and pricing structure. In *Proc. 7th Int. Conf. on Telecommunications Systems: Modeling and Analysis*, pp. 254–267.

Bass, F. M. 1969 A new product growth model for consumer durables. *Management Sci.* **15**, 215–227.

Bass, F. M. 1980 The relationship between diffusion rates, experience curves and demand elasticities for consumer durable technological innovations. *J. Business* **53**, 51–67.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Cerf, V. G. 1998 Interview. *Technos Q.* **7**(2), 4–9 (http://www.technos.net/journal/volume7/2cerf.htm).

Crowe, J. 2000 Silicon economics II: supply, demand, and disaggregation. Talk given at Level3 Analyst Conference, New York, 24 January 2000 (http://www.level3.com/US/SE/1,1716us|investor||statement,00.html).

Fletcher, R. 1987 *Practical methods of optimization*, 2nd edn. Wiley.

Freidenfelds, J. 1981 *Capacity expansion: analysis of simple methods with applications*. Amsterdam: North-Holland.

Gill, P. E., Murray, W. & Wright, M. H. 1981 *Practical optimization*. Academic.

Greene, W. 1993 *Econometric analysis*, 2nd edn. London: Collier-Macmillan.

HAI Consulting Inc. 1998 *HAI model description*. Release 5.0. Distributed by FCC.

Herk, L. F. 1993 Consumer choice and Cournot behavior in capacity-constrained duopoly competition. *Rand J. Economics* **24**, 399–417.

Lanning, S. G., Mitra, D., Wang, Q. & Wright, M. H. 2000 Modeling and optimization of large-scale networks. Research Memorandum, Bell Laboratories, Murray Hill, NJ.

O'Donnell, S. 1973 *Historical statistics of the electric utility industry through 1970*. New York: Edison Electric Institute.

Stern, T. E. & Bala, K. 1999 *Multiwavelength optical networks—a layered approach*. Addison-Wesley.